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Magnetohydrodynamic Power Extraction from Cold Hypersonic Airflows with External Ionizers

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A novel concept of hypersonic cold-air magnetohydrodynamics (MHD) power generators with ionization by electron beams is analyzed. Electron beams are shown to allow control and stable operation of MHD channels in cold high-speed flows. To avoid excessive energy cost of ionization and damage to beam-injection foils, electron beam current densities should be restricted to a few milliamperes per square centimeter. This reduces the conductivity in electron beam sustained MHD channels compared with that in conventional MHD generators, restricting performance and calling for very strong magnetic fields and high Hall parameters. The high Hall parameters cause ion slip and near-anode phenomena to become first-order issues. Example one-dimensional calculations of hypersonic power generator performance appear to be promising. Possible problems that could be caused by hypersonic boundary layers and electrode sheaths, including anode sheath instability and ways to avoid it, are also discussed.

I. Introduction

OVER the last several years, new applications of magnetohydrodynamic (MHD) technology to hypersonics have been proposed. One new concept, called AJAX, was suggested in Russia about 20 years ago.[1,2] A key element of AJAX is a MHD power generator at the inlet of the airbreathing hypersonic engine. The generated electricity could be used to power various electromagnetic devices onboard, or to provide a MHD acceleration of the engine exhaust flow (the so-called MHD bypass).

The MHD generator would reduce the total enthalpy of the flow at the engine inlet. If the MHD-reduced total enthalpy becomes low enough, a conventional turbojet or ramjet engine, instead of a scramjet, could be operated in hypersonic flight. Because of a certain flexibility of MHD devices, especially those with external ionizers (discussed subsequently), engine inlet flow parameters could be tailored to optimize engine performance in off-design environments.

Even if the MHD enthalpy extraction is not enough to avoid the need for a scramjet at very high Mach numbers, the MHD bypass could still be beneficial. As demonstrated in the recent theoretical papers,[3,4] MHD bypass could bring the scramjet inlet temperature to a tolerable level and, at the same time, increase the specific impulse.

The practicality of the MHD bypass concept depends on many system issues, such as cost, robustness, thrust-to-weight ratio, etc. To address these issues adequately, the physical feasibility of operating hypersonic MHD channels and the extraction of significant fractions of enthalpy with tolerable levels of losses must be considered. It is the purpose of this paper to analyze physical aspects of MHD power generation in hypersonic flows up to Mach 6–8.

The first challenge in developing onboard hypersonic MHD generators is to create an adequate electrical conductivity in air. With air entering the MHD channel at a static pressure of about 0.1 atm and temperature of a few hundred Kelvin (the static temperature could reach about 1000–2000 K at the channel exit), thermal ionization of even cesium-seeded gas is far below that required for MHD operation. When the freestream Mach number is very high, above Mach 12, the flow could be put through a series of oblique shocks, increasing static temperature to 2800–4000 K, at which point a conventional MHD generator could in principle be operated with 0.1–1% of potassium or cesium seed. This configuration is suggested and analyzed in recent NASA Ames Research Center studies.[3,4] Of course, at hypersonic speeds mixing of the injected seed with the flow is problematic. Additionally, thermal ionization in hot hypersonic boundary layers would be much higher than in the core flow, which could result in short circuiting through the boundary layer. Nevertheless, conventional MHD operation is at least conceivable at the very high flight Mach numbers.

The situation is more complicated at Mach numbers of about 8 and lower. Even after a few oblique shocks, static temperature would not reach the 2800–4000 K level required for conventional seeded MHD operation. Cold hypersonic MHD channels would, therefore, have to rely on a nonthermal ionization mechanism. As shown in Refs. 5 and 6, high-energy electron beams represent the most energy efficient way of creating ionization in cold gas. (The ionization efficiency is two to three orders of magnitude higher than that of electric field-sustained nonequilibrium ionization).

In Refs. 5 and 6, we described, qualitatively and quantitatively, propagation of electron beams and the ionization profiles created by them in air. We also analyzed, in a quasi-one-dimensional approximation, theoretical performance of Faraday-type generators and accelerators sustained by electron beams. Our preliminary analysis[4–7] of the role of boundary layers and electrode sheaths indicated that the layers and sheaths are strongly coupled with each other and can play a critical role in hypersonic MHD channels.

In this paper, we further develop ideas and methods presented in Refs. 4–7, focusing on electron beam-sustained hypersonic MHD generators. First, we discuss basic requirements and constraints on ionization level, magnetic field, and channel length. Then, one-dimensional analysis of Hall-type MHD generators with electron-beam ionization is performed. Electrode sheaths are considered next. We demonstrate that anode sheaths play a crucial role and are subject to a potential instability; we also indicate ways to suppress the instability.

II. Electron-Beam Ionization in MHD Channels: Basic Configuration and Potential Advantages

As discussed in Refs. 5 and 6, high-energy electron beams are the most efficient ionization sources. High-energy beam electrons would collisionally ionize atoms and molecules with very high

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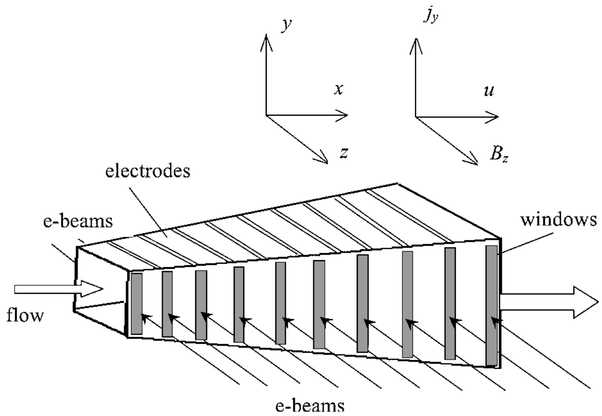

Fig. 1 Schematic of an MHD channel with electron beam ionization.

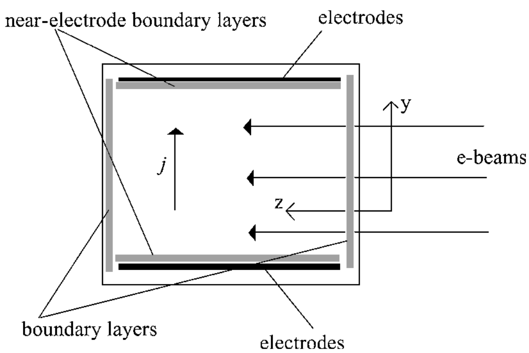

Fig. 2 Schematic of an MHD channel with electron beam ionization: side view.

probability, and secondary electrons would cause additional ionization. Thus, a cascade of electrons is generated as the beam slows down in the gas. The net effect is approximately one ionized molecule or atom for every $Y_i = 35$ eV lost by the beam. This ionization cost is only 2–3 times greater than the ionization energy of air molecules.

The basic configuration of an MHD channel with electron-beam ionization is shown in Figs. 1 and 2. An array of segmented electrodes allows a transverse electric current to flow through the gas. The magnetic field, orthogonal to both electric field and the flow direction, produces the accelerating or decelerating $\boldsymbol{j} \times \boldsymbol{B}$ force (in Figs. 1 and 2, the accelerator case is shown). Total flow enthalpy is affected not only by the $\boldsymbol{j} \times \boldsymbol{B}$ force, but also by the inevitable Joule heating and by the energy input from the ionizing beams. The beams, as shown in Figs. 1 and 2, have to be injected along magnetic field lines because the electrons tend to spiral around the field lines, and calculations show that the Larmor radius is much less than a millimeter at magnetic fields of several tesla and higher.

To extract an enthalpy of a few megajoules per kilogram from a high-speed (Mach 6–8) airflow at densities on the order of 0.1 atmospheric density, over a reasonable length, 1–4 m, an electron number density at least on the order of 10^{12}–10^{13} cm^{-3} is required.[8,9] Estimates performed in Refs. 5, 6, 8, and 9 indicate that this electron number density could be sustained by an array of electron beams with energy of 10–50 keV and beam current density of between 1 and 100 mA/cm^2.

As a practical matter, it would be very hard if not impossible to have electron beams fill out the entire sidewall of the channel more or less uniformly. Indeed, the beams would be generated in vacuum and would have to be injected into the channel through some foils or windows. The windows or foils would be heated by the beams passing through and also would have to be of sufficient mechanical strength. If the beam current density is on the order of 1 mA/cm^2 or less, thin metallic foils with passive cooling should suffice.[10] For higher beam current densities, either active cooling or the new technology of so-called plasma windows or ports[11,12] would have to be utilized. In any case, electron beams will be passed through only a part of the sidewall, and Fig. 1 shows the pass-through foils or windows as a periodic array of slots.

The rate of energy loss by the beam electrons is fairly independent of energy if the electron energy is greater than a few hundred kiloelectron volts (Refs. 10 and 13). However, in the 30-keV range, the energy loss per unit length traversed increases strongly as the electron energy decreases. Thus, the energy deposition per unit length by the beam is not uniform across the MHD channel. In addition, the electrons tend to follow the magnetic field across the channel because any attempt to travel perpendicular to the field lines results in a helical path due to the Lorentz force.

Modeling of electron-beam-generated ionization profile across the channel was described in Refs. 5 and 6. To conclude this section, we list some of the advantages of the ionization by electron beams.

1) Electron beams represent the most efficient way of ionizing cold gases. The energy cost of ionization, $Y_i = 35$ eV, is three orders of magnitude lower than that in electric field-sustained discharges.

2) Because for high electron energies, ionization cross sections increase when the incident electron energy decreases, beams injected from the sidewall would generate higher ionization inside the channel than near the wall. Thus, the problem of short circuiting through the hot, highly conductive, boundary layer, limiting performance of conventional MHD devices, could be minimized in devices with electron-beam ionization.

3) Plasmas with externally sustained ionization are inherently more stable than those of self-sustained nonequilibrium discharges. Indeed, ionization generated by electron beams is essentially decoupled from both temperature and electric field. A local positive fluctuation in temperature that would have led to arcing instability in self-sustained discharges would result in reduced ionization by electron beams and reduced local heating, stabilizing the plasma.

III. Constraints on Ionization Level, Magnetic Field, and Channel Length: Role of Ion Slip

For better operation of an MHD channel, electrical conductivity, proportional to the ionization fraction, should be high. However, when the conductivity is created by an external ionizer, such as an electron beam, higher conductivity means that more energy will have to be spent on ionization. As mentioned earlier, the energy cost of an ion–electron pair created by a high-energy electron beam is $Y_i = 35$ eV. Because at steady state the ionization rate is balanced by the rate of recombination, the minimum power per unit volume required to sustain electron density n_e is $k_{\mathrm{dr}} n_e^2 Y_i$, where $k_{\mathrm{dr}} \approx 10^{-7}$ cm^3/s is the rate constant of dissociative recombination. This power has to be substantially lower than the power extracted from the flow, which can be, per unit volume, estimated roughly as $\frac{1}{2}\chi \rho u^3 / L$, where χ is the enthalpy extraction fraction, ρ is the gas density, u is the flow velocity, and L is a characteristic length on the order of the channel length. The criterion

$$n_e < u \sqrt{\frac{\chi \rho u}{2 L k_{\mathrm{dr}} Y_i}} \tag{1}$$

with the density of about 0.1 normal atmospheric density, velocity corresponding to Mach 6–8 flight, enthalpy extraction fraction $\chi \approx 0.2$–0.5, and L-3 m (see subsequent discussion), requires that n_e be not higher than about 10^{12} cm^{-3}. At pressures of 0.01–0.1 atm, this results in electrical conductivity on the order of 0.1–1 mho/m, one or two orders of magnitude lower than typical conductivity in conventional MHD channels.

With the conductivity that low, appreciable MHD effects can be obtained only in very strong magnetic fields, at least several tesla, preferably 10–20 T. An immediate consequence of having very strong magnetic fields at low gas density is that electron Hall parameter Ω_e defined as the ratio of electron cyclotron frequency and the electron-neutral collision frequency, becomes very large, 10 and greater.

No conventional MHD channel operates at such high Hall parameters. A possibility of operating a hypersonic MHD channel with electron-beam-sustained ionization at very high Hall parameters is a very challenging issue. One effect that has to be taken into account is the so-called ion slip.[14,15] To estimate the magnitude of this effect, consider basic physics of MHD deceleration of weakly ionized gas.

When a weakly ionized gas enters an MHD generator, Faraday emf causes electric charges to move in the transverse direction, and the resulting Lorentz force decelerates the plasma. Because electrons are very light, they experience the deceleration first, whereas ions, due to inertia, try to move at constant velocity. However, even a slight separation of ions and electrons generates an ambipolar electric field. This field glues electrons and ions together, so that they move as a whole, maintaining quasi neutrality and experiencing deceleration together. (It can be demonstrated that the ambipolar field is exactly equal to the axial Hall field.) To decelerate the weakly ionized gas, its neutral molecules must be affected. The molecules cannot directly experience emfs, and they could be decelerated only in collisions with ions and electrons. Those collisions manifest themselves macroscopically as a friction force. When velocities of electrons and ions relative to the neutrals are equal, electrons, due to their very light mass, carry only a very small momentum relative to the neutrals. Therefore, it is the slip, or velocity difference, between ions and molecules that transfers the Lorentz force to the molecules.

The transverse electric current in the channel can be expressed as

$$j_y = \alpha \cdot \sigma u_i B \tag{2}$$

where α is a numerical factor between 0 and 1 (for a Faraday generator, $\alpha = 1 - k$, where k is the loading factor), σ is the electrical conductivity, u_i is the ion (and electron) axial velocity, and B is the magnetic field. At quasi-steady state, the Lorentz force $j_y B$ is equal to the molecule–ion friction force per unit volume:

$$j_y B = \alpha\sigma u_i B^2 = k_{\text{in}} n_i n m_n (u - u_i) \tag{3}$$

where k_{in} is the rate coefficient of ion–molecule collisional momentum transfer; n, m_n, and u are number density, mass, and axial velocity of molecules; and n_i is the ion number density. The conductivity can be written as

$$\sigma = e^2 n_e \big/ m k_{\text{en}} n \tag{4}$$

where n_e and m are the number density and mass of electrons, and k_{en} is the electron–molecule collision rate coefficient. It is also convenient to introduce electron and ion Hall parameters:

$$\Omega_e = eB/mk_{\text{en}}n, \qquad \Omega_i = eB/m_n k_{\text{in}} n \tag{5}$$

The ratio of the ion and electron Hall parameters is

$$\Omega_i/\Omega_e = k_{\text{en}} m / k_{\text{in}} m_n \approx \sqrt{m/m_n} \ll 1 \tag{6}$$

From Eq. (3), using definitions (4) and (5), we obtain the relation between the ion–electron velocity and the gas velocity:

$$u_i = u/(1 + \alpha\Omega_e\Omega_i) \tag{7}$$

When the electron Hall parameter is not very large,

$$\Omega_e < 1\big/\sqrt{\alpha}\ \sqrt[4]{m_n/m} \tag{8}$$

the ion–electron and gas velocities are close to each other. However, as magnetic field strength increases, the product in the denominator of Eq. (7) becomes larger than 1, and ions appreciably slip against molecules. Qualitatively, ion slip reduces performance of MHD generators because the velocity u_i in the expression for the Faraday emf, $u_i B$, is smaller than the gas velocity u. A simple recipe for including the ion slip effect into a formal set of MHD equations[14,15] is to divide both conductivity and Hall parameter by the factor $(1 + \Omega_e\Omega_i)$ (see the next section).

With performance reduction by the low ionization level and the ion slip effect, the channel length necessary for a substantial enthalpy extraction could get very large. The characteristic length L needed to substantially reduce the flow enthalpy can be estimated from energy balance:

$$\tfrac{1}{2}\rho u^2 \approx j_y B \cdot L = \alpha\sigma u_i B^2 L \tag{9}$$

Taking into account formulas (4), (5), and (7), we obtain the estimate

$$L \approx \frac{1}{2} u \tau_i \cdot \frac{1 + \alpha\Omega_e\Omega_i}{\alpha\Omega_e\Omega_i} \tag{10}$$

where

$$\tau_i = 1/k_{\text{in}} n_i = 1/k_{\text{in}} n_e \tag{11}$$

is the mean time for a molecule to experience a collision with ion.

The physical meaning of the estimate (10) is transparent. The minimum time that a gas particle needs to experience MHD deceleration is simply the time a molecule needs to experience at least one collision with an ion. Therefore, the minimum length of the channel is roughly the flow velocity multiplied by the τ_i (or Mach number of the flow multiplied by the molecule mean free path with respect to collisions with ions). The required channel length is minimal at very strong magnetic fields, when

$$\frac{1 + \alpha\Omega_e\Omega_i}{\alpha\Omega_e\Omega_i} \to 1 \tag{12}$$

As magnetic field strength decreases, when $\alpha\Omega_e\Omega_i \ll 1$, the required channel length increases as $(\alpha\Omega_e\Omega_i)^{-1}$.

Numerical estimates with formula (10) show that if the density of electrons and ions reaches 10^{14} cm^{-3}, typical for conventional seeded MHD generators, the length L stays within a few meters even at electron Hall parameters of about 3 and loading parameter k of about 0.9. In contrast, the low electron number densities (10^{12} cm^{-3}) in electron-beam-sustained channels require electron Hall parameters above 10 to extract a substantial amount of enthalpy in 3–4 m.

The need to have very large Hall parameters makes operation of a Faraday channel problematic. In a Faraday channel, allowing a longitudinal Hall current to flow would sharply reduce performance. The Hall current could be prevented, in principle, by segmenting electrodes. In an ideal Faraday channel, performance is the same as it would be in the absence of Hall effect. However, even if the Hall current is eliminated in the core flow, the problem exists in the boundary layer. There, the gas velocity is low, resulting in an unopposed Hall electric current flowing along the anode electrodes. Because the gas density in hypersonic boundary layers is much lower than in the core flow, electron Hall parameters in the boundary layer could reach 10^2–10^3. As a consequence, electric current near the anode would flow almost parallel to the anode electrodes, and electrons would enter the anode near its edge, where the electric field is the strongest. The current concentration near anode edges, separated by insulation, would lead to arcing and instabilities and could seriously impede performance of the Faraday channel.

Thus, it could be beneficial to use Hall configuration, where opposing pairs of segmented electrodes are short circuited, and the longitudinal Hall current is collected for power generation. In addition to being a natural choice at very large Hall parameters, and the simplicity of having only one electric load, the Hall generator could help solve the electrode-edge current concentration problem. Indeed, because of the strong short-circuit longitudinal current, electrons would enter anodes almost normally to the anode surface, and only a small fraction of the current would concentrate at the electrode edge.

One important disadvantage of Hall generators is that it is hard to collect the current flowing along the wide channel. A partial solution to the second problem is provided by using the so-called diagonal connection,[14] which is close in performance to the Hall configuration at large Hall parameters, but makes the task of current collection easier. In this paper, we will analyze only Hall-type (or diagonal-connection) generators.

IV. Quasi-One-Dimensional Modeling of MHD Generators with Electron-Beam Ionization

A. Basic Equations

In the case of an ideal Hall power-generation channel, the set of equation is similar to the set of equations used in Refs. 5 and 6 for an ideal Faraday generator [Eq. (28) in Ref. 5 or Eq. (28) in Ref. 6]:

$$\rho u A = \text{const}, \qquad \rho u \frac{\mathrm{d}u}{\mathrm{d}x} + \frac{\mathrm{d}p}{\mathrm{d}x} = -j_y B_z$$

$$\frac{\rho u \mathrm{d}(\gamma\varepsilon + u^2/2)}{\mathrm{d}x} = -|j_x E_x| - Q_v + \frac{E_v - E_v^0(T)}{\tau_{VT}(T)} + Q_b$$

$$\frac{\mathrm{d}E_v}{\mathrm{d}x} = \left[Q_v - \frac{E_v - E_v^0(T)}{\tau_{VT}(T)}\right]\Big/ u + \frac{E_v}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x}$$

$$p = R\rho T = (\gamma - 1)\rho\varepsilon \tag{13}$$

Here $A = A(x)$ is the MHD channel cross-sectional area. In addition to continuity, momentum, and energy equations, this set includes the vibrational energy equation.

Nonequilibrium and equilibrium vibrational energy can be expressed through the respective temperatures (T_v is the vibrational temperature) by the Planck formula:

$$E_v = N\varepsilon_v = N\frac{\hbar\omega_0}{\exp(\hbar\omega_0/T_v) - 1}$$

$$E_v^0 = N\varepsilon_v^0 = N\frac{\hbar\omega_0}{\exp(\hbar\omega_0/T) - 1} \tag{14}$$

where $\hbar\omega_0$ is the lowest vibrational quantum of nitrogen molecules and the temperatures are expressed in energy units.

The vibrational excitation term Q_v can be expressed as a fraction η_v of the Joule heating rate:

$$Q_v = \eta_v\left(j_y^2 + j_x^2\right)\big/\sigma \tag{15}$$

The fraction η_v is a function of the effective reduced electric field E_{eff}/N where N is the number density of gas molecules. The fraction η_v was taken from Ref. 16, where it has been tabulated as a function of E/N on the basis of the solution of Boltzmann kinetic equation for plasma electrons. Nitrogen vibrational relaxation time τ_{VT}, in seconds, is sensitive to both temperature and the concentration of atomic oxygen, and it was taken as in Refs. 5 and 6:

$$1/\tau_{VT} = N \cdot \left\{7 \times 10^{-10} \exp\left[-\left(141/T^{\frac{1}{3}}\right)\right] + (N_{\mathrm{O}}/N) \cdot 5 \times 10^{-12} \exp\left[-\left(128/T^{\frac{1}{2}}\right)\right]\right\} \tag{16}$$

where N_{O} and N are the number density of oxygen atoms and the total number density of gas molecules, respectively, expressed in cubic centimeters per second. The values of the atomic oxygen mole fraction are discussed in Sec. IV.B.

Total power extracted from the MHD channel,

$$W_{\text{ext}} = \int_V |j_x E_x| \,\mathrm{d}V$$

and the efficiency, $\chi = W_{\text{ext}}/[\rho u A(u^2/2 + c_p T)]_{x=0}$, can be calculated after solving the gasdynamic equations.

According to Ref. 14, to take into account the ion slip effect, the conductivity and Hall parameter in MHD for quasi-neutral plasma flow should be substituted by

$$\sigma \rightarrow \Sigma = \sigma/(1 + \Omega_e\Omega_i), \qquad \Omega_e \rightarrow \tilde{\Omega} = \Omega_e/(1 + \Omega_e\Omega_i) \tag{17}$$

In this case equations for the currents j_x and j_y take the following form[14]:

$$j_x = \Sigma E_x^* - \tilde{\Omega} j_y, \qquad j_y = \Sigma E_y^* + \tilde{\Omega} j_x \tag{18}$$

where $E_{x,y}^*$ are the electric field components in the reference frame moving with the gas.

However, simple but cumbersome derivations of electron and ion currents and Joule dissipation rates, performed by D. Bogdanoff, C. Park, and U. Mehta, and verified by the present authors, show that in Eq. (15) for Q_v the substitution (17) should not be performed, and the conductivity σ in Eq. (15) has to be left uncorrected.

Current density and electric field need to satisfy the following conditions in a steady-state MHD channel:

$$\nabla \cdot \boldsymbol{j} = 0, \qquad \nabla \times \boldsymbol{E} = 0 \tag{19}$$

In our case, ionization by electron beams occurs in narrow layers, and between the layers we have a motion of decaying plasma (Fig. 1). Therefore, electron density and conductivity vary along the flow. In the one-dimensional model, the transverse component of the current, j_y, does not depend on y. Therefore, because of continuity of the total current, the longitudinal current is $I_x = j_x(x)S(x) = j_x(0)S(0) = \text{const}$. Thus, the longitudinal current density in an arbitrary selected cross section $S(x)$ is

$$j_x(x) = I_x/S(x) \tag{20}$$

Longitudinal current at the channel inlet is found from the standard one-dimensional model[14]:

$$j_x(0) = \left.\frac{\Sigma\tilde{\Omega}uB(1-\zeta)}{1 + \tilde{\Omega}^2}\right|_{x=0}, \qquad \zeta < 1 \tag{21}$$

The transverse field E_y^* in Eq. (19) is an effective transverse emf per unit width. In the present one-dimensional model, as in the case of uniform volume ionization in conventional Hall MHD generators,

$$E_y^* = -uB \tag{22}$$

When Eqs. (20) and (22) are taken into account, longitudinal field E_x^* and transverse current density j_y can be easily determined from the set of Eqs. (18).

Parameter $\zeta < 1$ is one of optimization parameters in our model.

In the standard Hall MHD generator model,[14] loading parameter k is defined as $k = |E_x/\Omega u B|$. In the present nonuniform case, we can define an effective loading parameter k,

$$k = \left|\frac{\int_0^{L_x} E_x^* \,\mathrm{d}x}{\int_0^{L_x} \tilde{\Omega} u B \,\mathrm{d}x}\right| \tag{23}$$

The plasma in both accelerator and power generator channels is modeled as consisting of electrons and positive and negative ions, whose number densities n_e, n_+, and n_- obey the quasineutrality $n_+ \approx n_e + n_-$. The set of equations for kinetics of charge species, accounting for electron beam-induced ionization rate (q_i term), attachment of electrons to molecules with formation of negative ions (frequency ν_a), collisional detachment of electrons from negative ions (rate constant k_d), and electron–ion and ion–ion recombination (rate coefficients β and β_{ii}, respectively), is

$$\frac{\partial n_e}{\partial t} + \frac{\partial \Gamma_e}{\partial x} = q_i + k_d N n_- - \nu_a n_e - \beta n_+ n_e$$

$$\frac{\partial n_+}{\partial t} + \frac{\partial \Gamma_+}{\partial x} = q_i - \beta_{ii} n_- n_+ - \beta n_+ n_e$$

$$\frac{\partial n_-}{\partial t} + \frac{\partial \Gamma_-}{\partial x} = -k_d N n_- + \nu_a n_e - \beta_{ii} n_- n_+ \tag{24}$$

For one-dimensional steady flow ($\partial n_{e,+,-}/\partial t = 0$), the fluxes of charged species can be written simply as $\Gamma_{e,+,-}(x) = n_{e,+,-} u_p$. In general, plasma velocity u_i differs from the gas velocity u due to the ion slip effect. Because of the quasi neutrality, there is a single electron–ion, that is, plasma, velocity in the laboratory reference frame. This plasma velocity, as follows from Eq. (3) (see also Ref. 15), is equal to

$$u_p = u_i \approx u - j_y B/n_+ m_n \nu_{\text{in}} \tag{25}$$

where m_n is ion mass and ν_{in} is the ion–molecule collision frequency.

The approximate formula for the electron-beam ionization rate is[5,6]

$$q_i = 2\varepsilon_b j_b / e Y_i Z \qquad (26)$$

Rate coefficients of electron–ion and ion–ion recombination, and of electron attachment and detachment processes, discussed in Refs. 5 and 6, were taken from Refs. 17 and 18. Because some of those rate coefficients depend on electron temperature T_e, it is important to calculate the electron temperature in the modeling. In our computations, electron temperature was determined from the tabulated data on electron diffusion and mobility coefficients of Ref. 16. In that paper, the diffusion and mobility coefficients are listed as functions of E/N, determined from experimental data and extrapolation based on solution of Boltzmann kinetic equation for electrons in air.

B. Estimates of Chemistry and Vibrational Relaxation

Oxygen atoms are produced by direct dissociation of oxygen molecules by high-energy beam electrons. Additionally, beam electrons dissociate nitrogen, and, if the gas temperature reaches 600–700 K, oxygen atoms are produced in the reaction $N + O_2 \rightarrow NO + O$. Because the energy threshold for dissociation is comparable with that for ionization, the beam-induced dissociation and oxygen atom production rate (in $cm^{-3} \cdot s^{-1}$) can be estimated as approximately equal to the ionization rate, which, for Mach 6–8 cases at 30-km altitude, ranges from 4×10^{17} down to about 5×10^{16} $cm^{-3} \cdot s^{-1}$. An additional channel of production of O atoms is dissociative attachment of low-energy plasma electrons: $e + O_2 \rightarrow O + O^-$. However, in all computed cases, attachment rate was at least an order of magnitude lower than the ionization rate (so that the ionization is balanced by electron–ion recombination). Therefore, the beam-induced ionization rate can serve as a good approximation for the O atom production rate.

O atoms could be destroyed in three-body processes such as $O + O_2 + M \rightarrow O_3 + M$, $O + O + M \rightarrow O_2 + M$, and $O + N + M \rightarrow NO + M$. However, with the gas number density of 10^{18} cm^{-3} or lower, these recombination processes would take at least 10 ms, whereas the flow residence time in 3-m-long MHD channel is only 1–2 ms. If nitrogen vibrational temperature gets as high as 3000–6000 K, the reaction $O + N_2 \rightarrow NO + N$ will become significant in removing O atoms. However, the reaction $N + O_2 \rightarrow NO + O$ will replenish O atoms. Thus, a reasonable estimate of O atom density can be done by putting their production rate equal to the beam-induced ionization rate and neglecting all chemical removal processes, so that the produced atoms simply move with the flow. Because the flow residence time in electron-beam ionized regions is about 1 ms, the relative concentration of oxygen atoms reaches $N_O/N \approx 10^{-4}$–10^{-3}.

With gas temperatures in the range 400–900 K and O atom density quite low, nitrogen vibrational relaxation time will be longer than 10 ms. Therefore, vibrational temperature will keep growing along the channel and could be quite high at the exit.

C. Results of One-Dimensional Modeling

One-dimensional modeling was performed for a number of cases, corresponding to a range of flight Mach numbers and altitudes. A representative case corresponds to Mach 6 flight at the altitude of 30 km. The flow was assumed to pass through an oblique shock formed by a 10-deg wedge upstream of the MHD channel entrance. The inlet cross section of the channel was taken as 25×25 cm^2 and the exit cross section as 75×75 cm^2. The channel length was fixed at 3 m. The magnetic field was assumed to be uniform and equal to $B = 7$ T, which is close to the current practical limit of superconducting magnets. Electron beams were assumed to be injected through periodically spaced slots from both sides of the channel, as shown in Fig. 1. The slot width was taken to be 0.5 cm, and the spacing between the slots was also 0.5 cm. The beam energy was adjusted according to the channel width and the gas density at the location. The current density of the beams was varied to give the best power extraction and the lowest stagnation temperature at the exit, while keeping the static pressure from increasing.

The set of Eqs. (13) and (24) at steady state was solved by the modified second-order Euler method. The number of equal spatial steps was $n \geq 1000$. Further increasing the number of steps was found to produce negligible variation of results.

The results are shown in Figs. 3–5. In Figs. 3–5, the freestream conditions are $M = 6$ and $h = 30$ km, with wedge angle $\alpha = 10$ deg. The inlet parameters are $T = 349.26$ K, $p = 4.39 \times 10^3$ N/m^2, $u = 1742$ m/s, and $M = 4.647$. The inlet and exit cross sections

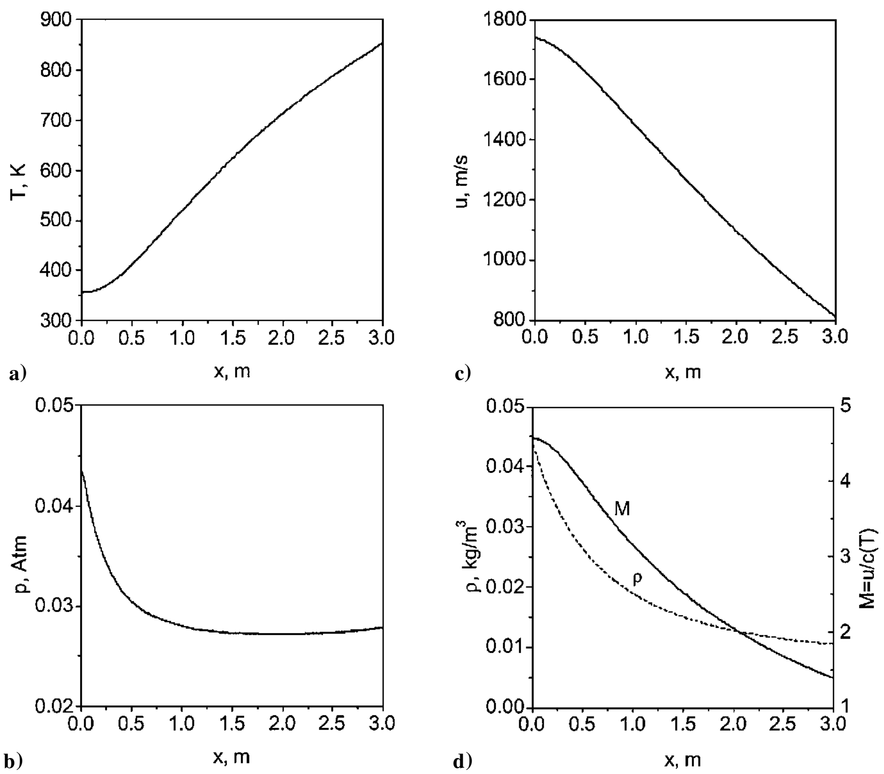

Fig. 3 Hall MHD channel profiles: a) temperature, b) static pressure, c) velocity, and d) Mach number and density.

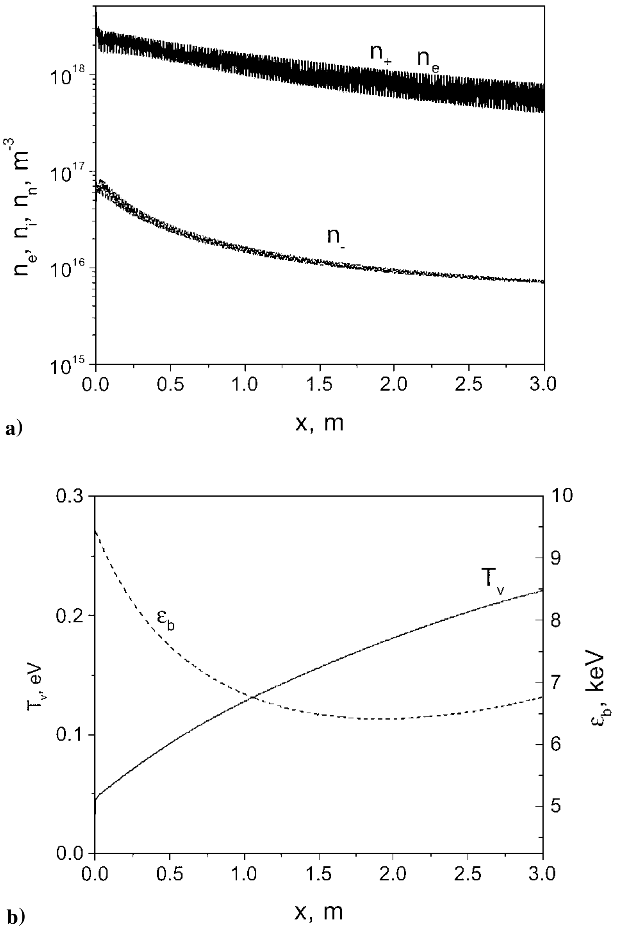

Fig. 4 Figure 3 case: a) plasma components densities and b) electron-beam energy and gas vibrational temperature.

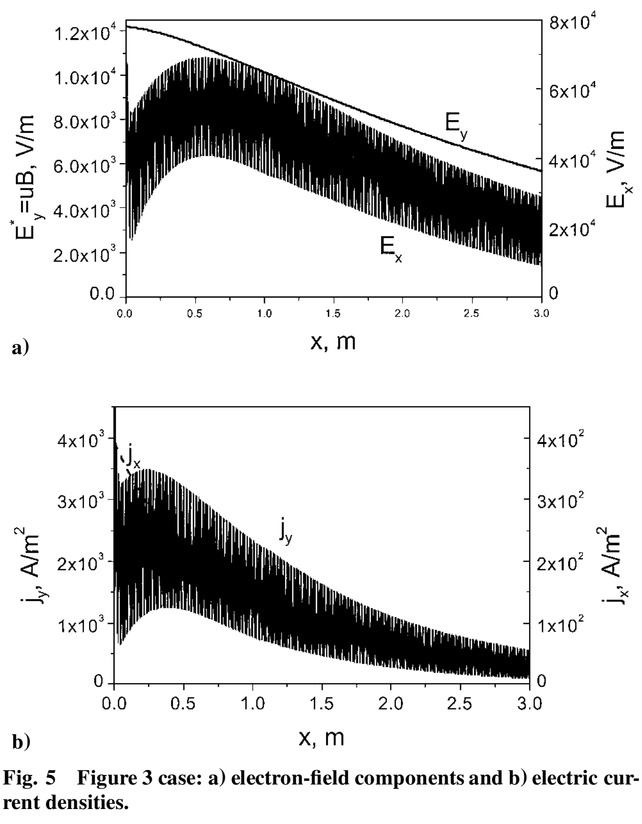

Fig. 5 Figure 3 case: a) electron-field components and b) electric current densities.

are $A_{\text{in}} = 0.25 \times 0.25$ m^2 and $A_{\text{out}} = 0.75 \times 0.75$ m^2. The magnetic field is $B = 7$ T. The electron-beam current density is $j_b(x) = 5 \times [z_{\text{in}}/z_x]^{1.15}$ mA/cm^2. The electron-beam injection slot width is 0.5 cm with spacing between the slots of 0.5 cm. The loading parameter is $k = 0.455$. The extracted power is $W_{\text{output}} = 2.92$ MW ($\chi = 32.9\%$ of inlet enthalpy flux). The power spent on ionization is $W_{\text{beam input}} = 237$ kW. The main result is that, for the chosen channel geometry, about 3 MW of electric power could be extracted from the flow, while spending only about 237 kW on ionization by electron beams. The extracted power is about 33% of the enthalpy flux into the channel.

As seen in Fig. 4, the gas is expected to be in a state of vibrational disequilibrium at the MHD channel exit. This is an effect of low-energy plasma electrons vibrationally exciting molecules in collisions, thus converting a part of flow energy into the energy of molecular excitation. Because of the low density and temperature, vibrational relaxation in the channel is slow, and excited molecules exit the channel virtually without quenching. The relaxation would occur in the diffuser or in the engine downstream of the MHD channel, where density and temperature increase.

One very important constraint on operating conditions of conventional Faraday MHD channels with thermal ionization of alkali seed, as pointed out in recent papers,[3,4] is that the longitudinal electric field should not exceed about 5000 V/m. If the longitudinal voltage gradient is higher, arcing between two neighboring electrode segments separated by insulation would start, reducing MHD performance and damaging the hardware. As seen in Figs. 3–5, in the computed case E_x exceeds the 5000 V/m, which can be a reason for concern. However, it is not clear whether the 5000-V/m empirical limit can be applied to Hall channels with electron-beam ionization. Gas temperature even in the boundary layer is much lower than the 3000–4000 K temperatures in conventional seeded channels. Therefore, thermal ionization is negligible, and electrical conductivity is controlled by electron beams, as in the core flow. Additionally, melting and sputtering of electrode and insulation material would certainly be less severe in our low-temperature cases. One has to bear in mind that Hall parameters in the boundary layer would be quite high (up to 30–50) in our cases, impeding electron mobility across magnetic field and thereby substantially increasing the voltage required for breakdown and sustaining arcs. Finally, arcing could have less of an effect on performance of Hall generators, where there already is a large longitudinal current, than on performance of Faraday channels. In any case, the issue of maximum longitudinal field that can be tolerated in cold Hall channels with electron-beam ionization should be addressed in future experiments.

Figure 6 summarizes performance predictions for various flight Mach numbers and altitudes. The channel geometry shown in Fig. 6 is as follows: inlet 25×25 cm^2, exit 50×50 cm^2 (for the altitude $h = 40$ km only) or 75×75 cm^2 (all other altitudes), and length $L = 3$ m. The magnetic field is $B = 7$ T. Upstream of the MHD channel, the air passes through an oblique shock on a wedge with the angle $\alpha = 10$ deg ($h > 15$ km) and $\alpha = 10$ or 5 deg ($h = 15$ km). The channel configuration may be either Hall or Faraday (at altitudes $h > 20$ km), but only Faraday at $h < 20$ km. The main result is that a large percentage of flow enthalpy, corresponding to several megawatts of power, could be extracted as electricity, while spending a relatively small fraction of the extracted electric power on electron beam ionization.

V. Anode Sheaths

As discussed in detail in Refs. 5–7, electrode sheaths are very important in hypersonic MHD channels with external ionization. The principal effect is that the same electric current that flows through the quasi-neutral plasma must be carried by positive ions in the cathode sheath and by electrons, in the anode sheath. Because ions have a low mobility, cathode voltage fall has to be large even in conventional glow discharges with cold cathodes. In hypersonic channels, cathode sheaths are immersed in boundary layers whose density profiles obviously affect the ion mobility, which affects the sheaths.

Of course, in a heated-cathode sheath, there is no need for high-voltage fall (as in conventional glow discharges) because heating

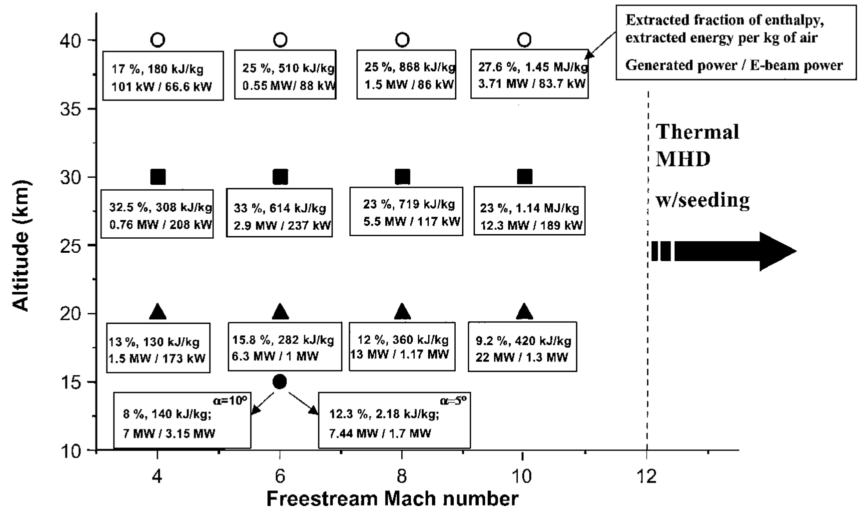

Fig. 6 Performance of electron beam supported MHD generators; numbers in boxes are generated power as a percentage of total enthalpy flux into the channels, generated power per kilogram of air, absolute power generated in the channel, and (after the slash symbol /) power spent on electron beam ionization.

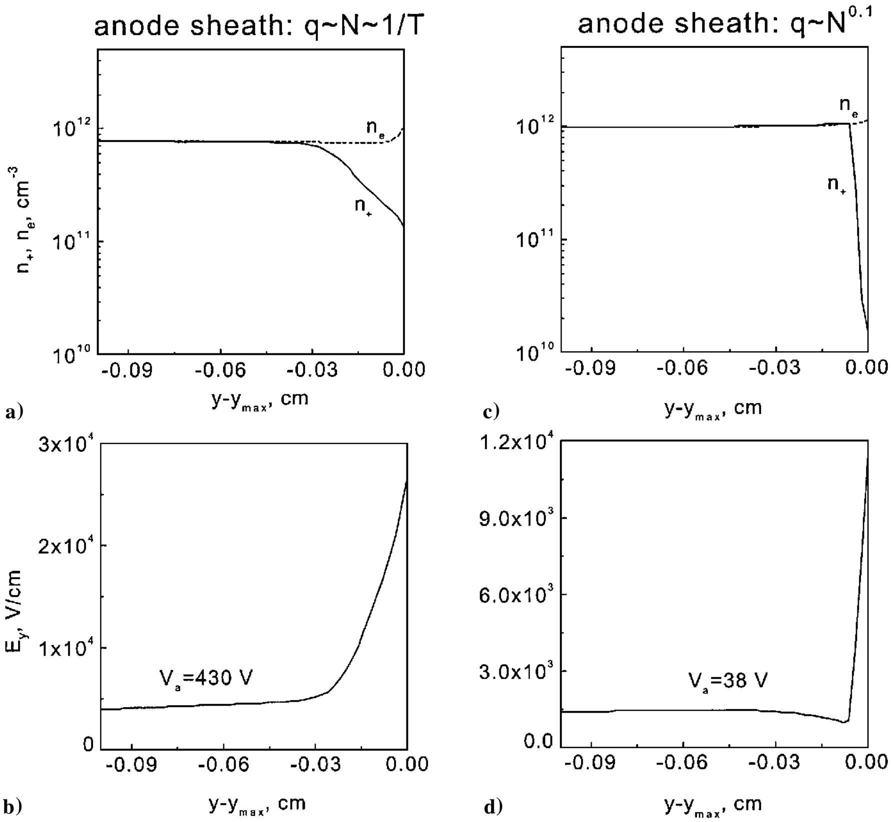

Fig. 7 Anode sheath structure at $x = 1$ m from the entrance to the Hall MHD channel: a) profiles of electron and ion densities when ionization rate is proportional to the gas density, b) electric field profile in the same case, c) profiles of electron and ion densities when ionization rate is close to that inside the channel due to increased electron beam current density and gas cooling, and d) electric field profile in the same case as c).

the cathode could provide an adequate thermal electron emission, thus supplying electrons and sustaining the current at small cathode voltage fall. For example, a Ba–Ni cathode with the surface temperature of $T \approx 900$ K can provide thermal emission current of $j \sim 1$ A/cm^2 (Ref. 19), which is quite sufficient to sustain the current in MHD channels considered in this paper. In hypersonic channels, heating the cathode to 800–900 K presents no problem: The cathode would just have to be cooled less because the boundary layer is quite hot.

The anode sheath (AS) is very different. In conventional glow discharges, the anode voltage fall is very low because the electrons in the sheath easily carry the current. In MHD channels, however, electrons have to drift through the AS across a strong magnetic field. Because electron mobility across the strong magnetic field is low, a strong electric field arises in the AS, and the anode voltage fall may become very large. To minimize the anode fall, it is critical to provide as high an ionization level in the AS as possible: The increased conductivity would result in a decrease in the AS voltage fall. In conventional MHD devices, ionization is thermal and is due mostly to seed atoms; therefore, in an AS immersed into the boundary layer, gas temperature is higher than in the core flow, increasing ionization rate. The AS in conventional channels, thus, hardly has a problem with supplying enough electrons to sustain the current, and the anode voltage fall is relatively small even in strong magnetic fields. In our case, ionization is done by electron beams injected into the flow, and the rate of ionization, that is, the number of ion–electron pairs generated each second per unit volume, is proportional to gas density. Hence, the ionization rate is lower in hot boundary layers than in the core flow. Additionally, the plasma in the AS is not quasi neutral, the ion slip effect all but disappears, and electron transport properties are determined by the full Hall parameter Ω_e instead of the reduced Hall parameter (17), so that the effective electron mobility across magnetic field is very low. Thus, to sustain the current, the anode voltage fall becomes very large. This increases Joule dissipation in the AS and, when the Joule dissipation becomes comparable with the viscous energy dissipation, $W_J = \boldsymbol{j} \cdot \boldsymbol{E} \sim W_\eta \sim (\eta + \eta_T)u^2/\delta^2$, could lead to an instability,

$$W_J \geq W_\eta \Rightarrow T \uparrow \Rightarrow N \downarrow \Rightarrow \Omega_e \sim 1/N \uparrow \Rightarrow E_y, V_A \uparrow \Rightarrow W_J \uparrow \Rightarrow \cdots$$

The instability chain can be broken by increasing the ionization rate in the AS to approximately that in the core flow and/or by cooling the gas in the AS. In fact, both cooling resulting in the density increase and increasing electron-beam current density in the AS would enhance ionization and stabilize the AS.

Figure 7 shows profiles of plasma parameters across the AS for the Mach 6, 30-km altitude case. The transverse current density is $j_y = 0.5$ A/cm^2. Boundary layers and, as an overlay, AS were computed with physical models and algorithms of Refs. 4 and 5. The anode was assumed to be cooled and, thus, maintained at constant temperature, so that the temperature in the boundary layer had a maximum at some point in the sheath. The AS was computed for the cross section 1 m downstream of the MHD channel entrance. Figures 7a and 7b correspond to ionization rate $q(y) = q(y_{\max})N(y)/N(y_{\max})$, that is, with no special cooling of the AS and with electron-beam current density equal to that in the core flow. Figures 7c and 7d correspond to ionization rate $q = q(y_{\max}) \cdot [N(y)/N(y_{\max})]^{0.1} \approx$ const, which, as described earlier, could be achieved by increasing the local electron-beam current density and by cooling the AS. As seen in the Figs. 7, the enhanced ionization rate can dramatically decrease the anode voltage fall and, thus, stabilize the anode sheath.

VI. Conclusions

In this paper, we analyzed physical aspects of a novel concept: cold-air hypersonic MHD power generators with ionization by electron beams. The analysis demonstrated that, with careful choice of parameters, electron beams can form stable, well-controlled plasmas, avoiding many shortcomings of conventional MHD channels and potentially enabling a good level of generator performance. At Mach 6–8, about one-quarter to one-third of the total flow enthalpy can be extracted as electricity. It is conceivable that, in a hypersonic flight, stagnation temperature in the engine could be reduced to about 1000 K, potentially enabling a turbojet operation.

Although electron-beam-generated nonequilibrium ionization in MHD channels may be technically feasible, there are a number of problems. To keep the fraction of extracted power that has to be spent on ionization at an acceptable level, electron density and conductivity would have to be at least an order of magnitude lower than those in conventional channels with thermal ionization of alkali seed. Because of the low conductivity, very strong magnetic fields are required for a substantial MHD effect. In strong magnetic fields and low-density gases, Hall parameters become extremely large, which makes operation of Faraday channels problematic due to possible arcing between anode segments. Hall or diagonal-connection configurations appear to be the natural choices at high Hall parameters. However, mostly because in these configurations pairs of opposing electrodes are shorted, there is an excessive heating and entropy generation in such channels. Calculations of entropy generation and thermodynamic efficiency are yet to be performed, but it is quite conceivable that the electron-beam-supported Hall generators produce more entropy than a series of oblique shocks.

A number of technical issues should be addressed before a final assessment of the nonequilibrium MHD concept. Among these issues are, for example, cooling channel walls, electrodes, and electron-beam pass-through windows; configuring ionization profiles; and stabilizing anode sheaths at high Hall parameters. When a physical feasibility is established, entropy analysis of the system's performance as compared to conventional designs should be done. Practical implementation of the concept also depends quite critically on availability of lightweight robust superconducting magnets with a large bore. Perhaps novel materials and construction methods would be required to make the magnets practical for use in hypersonic MHD devices.

Most of the technical issues could be resolved in experimental studies of MHD channels with ionization by electron beams.

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